Digital Compensation of Second- and Third-Order Nonlinear Distortions Generated by Blocker Signals

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Deeply integrated systems in chips commonly include a digital and an analog front end on the same die. These analog front-end schemes for wireless communications could be implemented under the concept called software-defined radio (SDR). Digital signal processing is commonly used to perform signal filtering and channel equalization, and, recently, to improve front-end radio performance by removing the undesirable effects of the analog front-end imperfections. These wide-band SDR are currently implemented without the surface acoustic wave (SAW) filter, because it is difficult to integrate a highly configurable one, as is required in wide-band systems. An analog front end without this filter has no efficient protection against blocker signal effects, specifically against nonlinear distortions due to the analog front-end imperfections. This paper proposes an algorithm to simultaneously remove second- and third-order nonlinear distortions caused by a blocker signal, departing from a behavioral model and a band-pass sampling pure digital algorithm to recover the blocker signal information.

Keywords: cross modulation, interference canceling, second- and third-order nonlinear distortions, band-pass sampling

1. Introduction

The integrated electronics industry began a revolution in all related areas, including wireless communications systems. This exponential growth has cost reduction as the main target: the principal trend is to integrate all subsystems into the same die. This means that any discrete component will be eliminated within the system. Taking a look at the communications circuits, a great evolution, enabled by the integrated electronics industry, can be seen. Deeply integrated radio frequency (RF) circuits with digital circuits in the same die can be commonly found. These technologies enable the expansion of the next-generation wireless communications systems [1], which are involved in two paradigms:(i) The cognitive radio, which was proposed in Ref. [2], enables a more efficient use of the RF spectrum, but requires smarter designs, specifically on RF front ends, to exploit all advantages of this. (ii) Software-defined radios are flexible and take advantage of digital signal processing to improve radio performance. Software-defined radio makes possible the cognitive radio.

Today, the RF front ends work in wide bandwidths, with some standard communications and with different modes of operation [3]. Commonly, modern radios use switches to select the subsys-tems to work in a defined operation mode. The implementation of the necessary components, for tuning in the RF circuitry, and the high integration effects produce performance degradation on the RF circuitry. The dirty radio approach [4] proposes that all undesirable effects in the analog front end could be compensated in the digital front end by means of pure digital signal processing. In the future, it is desirable that RF with tuning capabilities will also have digital signal processing capabilities to compensate any hardware imperfection effects [5]. Specifically, it could be very convenient to use digital signal processing to clean some kind of undesirable distortions [6]. However, very complicated relation-ships are found when trying to mitigate several imperfections at the same time (e.g., nonlinear response, IQ imbalance, DC offset).

A nonlinear receiver is a common result of hardware imperfec-tions with harmful effects in wireless systems. These imperfections become critical for the current designs without a surface acoustic wave (SAW) filter since they do not have any protection against unwanted out-of-band signals, specifically against the high-power ones called *blocker signals* [1,7,8]. Blocker signals are harmful if the desired signal is too weak with respect to them. In these con-ditions, baseband intermodulation products appear as a result of the analog front-end imperfections, which have power comparable with that of the signal of interest.

In the literature, some digital signal processing techniques have been proposed to compensate these kinds of distortions. A pioneering work in digital post-distortion was introduced in Ref. [9], which proposed a digital post-distortion to remove the co-channel interference. However, the first ideas for canceling cross-modulation interference were presented in Ref. [10], and subsequent publications improved this primary idea. The algorithm originally proposed in Ref. [10] is limited to the bandwidth defined by the operating Nyquist frequency. Taking into account that in practical systems the sample rate cannot be infinite and that faster sampling implies more power consumption, to increase the sample rate to include the blocker information is not possible in some scenarios, especially in applications with power consumption restrictions. In Ref. [1],it is suggested that the blocker information could be obtained by the addition of a downconversion circuit. Unfortunately, that solution becomes impractical according to the observed tendency in deeply integrated circuits to eliminate any analog circuitry. Some other related works could be found in the literature, e.g. Refs [11–14], however, but none has proposed an efficient and practical way to obtain blocker signal information.

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The compensation technique proposed in this paper obtains the blocker signal information by means of subsampling (i.e., a simple decimation procedure), namely pure digital signal processing of the received signal without additional downconversion circuitry. The idea is to take advantage of the subsampling properties to recover the blocker information by moving a blocker alias spectrum within the operation bandwidth. With the proposed algorithm, it is possible to compensate the corresponding second and third order nonlinear distortions introduced within the baseband by the blocker. To our knowledge, in the state-of-the-art literature there is no other pure digital technique capable of mitigating nonlinear distortions due to blocker signals located beyond the operation Nyquist frequency.

The organization of the rest of the paper is as follows. In Section 2 the system model is presented. In Section 3 the algorithm to recover the blocker information is proposed. In Section 4 the algorithm to cancel the second and third nonlinear distortions is introduced. In Section 5 the computer simulations results are reported. Finally, in Section 6 our conclusions are drawn.

Throughout this paper, the following notations are adopted. The notation $(\cdot)^*$ represents the matrix conjugate transpose. *floor*() returns the integer part of a real positive number. $Re\{\cdot\}$ and $Im\{\cdot\}$ return the real and the imaginary part of its arguments, respectively. $E\{\cdot\}$ is the expected value with respect to the underlying probabil-ity measure, and $diag\{\cdot\}$ represent the diagonal matrix determined by the vector in its arguments. Finally, $(\cdot) \star (\cdot)$ denotes convolution of its arguments, and bold faced letters represent the matrices.

2. System Model

Some sources of nonlinear distortion can be identified in an RF analog front end. The low-noise amplifier (LNA) is the principal source of third-order distortion products; some of these products may be harmful when a strong interference signal is received. On the other hand, the downconversion stage is the principal source of second-order distortion products. In a direct conversion receiver, these products could be dangerous and the performance of second-order products rejection could be degraded by some imbalance in the mixing core [15]. In the literature, one can find some techniques to improve second-order products rejection by using tuning nodes to compensate the imbalance, e.g. Refs. [16,17]. Digital compensation could also help to improve the tuning node limitations, especially in the presence of a blocker signal. The following subsections present the system model used in this paper. In Section 2.1 the nonlinear distortion caused by the LNA is analyzed. In Section 2.2 the analysis of the nonlinear distortion caused by the downconversion stage is presented. Finally, in Section 2.3 a system model considering both sources of nonlinear distortion is proposed.

2.1. Nonlinear distortion caused by the low-noise amplifier For analysis purposes, a polynomial model without memory is proposed. According to Ref. [10], a good approximation of the LNA's nonlinear behavior can be expressed by

$$y_p(t) = \alpha_1 v(t) + \alpha_2 v(t)^2 + \alpha_3 v(t)^3 + \omega_p(t) \qquad (1)$$

where $v(t)$ and $y_p(t)$ are the input and output signals, respectively; $\omega_p(t)$ is an additive Gaussian noise; and α_1, α_2, and α_3 are real constants. α_1 is the linear gain of the LNA, and α_2 is related to $IP2$ (second-order interception point) by

$$IP2 = \frac{|\alpha_1|}{|\alpha_2|}, \qquad (2)$$

and α_3 is related to $IP3$ (third-order interception point) by

$$IP3 = \sqrt{\frac{4|\alpha_1|}{3|\alpha_3|}}, \qquad (3)$$

which could be expressed as a function of the 1-dB compression point. Typically, the 1-dB compression point is about 10 dB below the $IP3$. The input signal $v(t)$ is defined by

$$v(t) = Re\{z_{l,0}(t)e^{j\omega_0 t} + z_{l,1}(t)e^{j\omega_1 t} + \ldots + z_{l,n}(t)e^{j\omega_n t}\}. \qquad (4)$$

where $z_{l,0}(t)$ is the received complex baseband envelope (chan-nel distorted), corresponding to the transmitted signal $x_{l,0}(t) = x_{l,0I}(t) + jx_{l,0Q}(t)$ centered at the frequency ω_0, and $z_{l,1}(t), \ldots, z_{ln}(t)$ are the complex baseband envelopes of the blocker sig-nals centered at frequencies $\omega_1, \ldots, \omega_n$, respectively. The rela-tionship between $z_{l,0}(t)$ and the transmitted signal $x_{l,0}(t)$ is defined by

$$z_{l,0}(t) = x_{l,0}(t) \star h_{l,0}(t), \qquad (5)$$

where $h_{l,0}(t)$ is a low-pass equivalent channel response. In this paper, the transmitted complex baseband envelope $x_{l,0}(t)$ is assumed to be an orthogonal frequency division multiplexing (OFDM) signal. For analysis purposes, it is also assumed that the received signal includes only the desired signal plus distortions generated by one blocker signal. Under this last consideration, the signal at the input of the LNA is defined by

$$v(t) = Re\{z_{l,0}(t)e^{j\omega_0 t} + z_{l,1}(t)e^{j\omega_1 t}\} \quad (6)$$

after LNA, where every component is going to produce self-distortion and inter-modulation products. The response of the LNA is given by

$$v_{lna}(t) = Re\{z'_{l,0}e^{j\omega_0 t} + z'_{l,1}e^{j\omega_1 t}\} + \omega_p(t), \quad (7)$$

where

$$z'_{l,0}(t) = \alpha_1 z_{l,0}(t) + \frac{3}{4}\alpha_3 z_{l,0}(t)|z_{l,0}(t)|^2 + \frac{3}{2}\alpha_3 z_{l,0}(t)|z_{l,1}(t)|^2, \quad (8)$$

$$z'_{l,1}(t) = \alpha'_1 z_{l,1}(t) + \frac{3}{2}\alpha'_3 |z_{l,0}(t)|^2 z_{l,1}(t) + \frac{3}{4}\alpha'_3 |z_{l,1}(t)|^2 z_{l,1}(t), \quad (9)$$

where α'_1 is the linear gain and α'_3 the third-order nonlinear gain on the blocker's channel. Some nonlinear products were neglected because they fall far from the band of interest and a band-pass filter is expected to be used after the LNA [18]. All nonlinear terms produced by the nonlinearity of LNA are shown in detail in Appendix A.

2.2. Nonlinear distortion caused by the downconver-sion stage The behavioral model of a mixer was introduced in Ref. [19]. The authors explain that the transconductance at the input stage is the principal source of nonlinear distortion because of the transconductance mixer widely used in direct-conversion radios. In fact, the input stage behaves as

$$i_{RF} = g_m[v_{in}(t) + \alpha_{2mix} v_{in}(t)^2 + \alpha_{3mix} v_{in}(t)^3 + \cdots], \quad (10)$$

where i_{RF} is the output current, g_m is the transconductance gain,
α'_2 and α'_3 are the nonlinear gains, and v_{in} (t) is the input voltage. It is well known that the odd-order nonlinear products are not as significant as the even-order ones in the mixer circuitry [20]; the analysis of this paper is focused on the following signal model:

$$i_{RF} = g_m[v_{lna} + \alpha_{2mix} v_{lna}(t)^2]. \quad (11)$$

Next, it is assumed that $g_m R = 1$, where R is the resistance load at the output of mixer, i.e. no gain in voltage-to-current and current-to-voltage conversion. As suggested in Ref. [21], the basic model
of a direct-conversion receiver is used (Fig. 1), where the mixing stage is simulated as a differential mixer. It is also assumed that there are imbalances in both the duty cycle of the local oscillator (LO) and the resistance loads. Since a perfectly balanced mixer has an infinite rejection of even-order products, the imbalances allow the even-order products to pass through a real mixer. Although the ideal waveform for the LO is assumed as perfect squared wave, in the real world it is difficult to produce. Instead, a high slew rate, in order to obtain the best conversion gain, is desirable.

A squared waveform with a duty cycle different from 50%is necessary to generate the switching pair imbalance. Thus, the squared LO waveforms, in each branch of the quadrature direct conversion mixer, are expressed by the Fourier series expansion as function of the duty cycle, that is

$$LO_{PI}(t) = \frac{1}{2} + \eta + \frac{2}{\pi}\sum_k G_1 cos(k\omega_0 t), \quad (12)$$

$$LO_{NI}(t) = \frac{1}{2} - \eta - \frac{2}{\pi}\sum_k G_1 cos(k\omega_0 t), \quad (13)$$

$$LO_{PQ}(t) = \frac{1}{2} + \eta + \frac{1}{\pi}\sum_k (G_2 cos(k\omega_0 t) + G_3 sin(k\omega_0 t)), \quad (14)$$

$$LO_{NQ}(t) = \frac{1}{2} - \eta - \frac{1}{\pi}\sum_k (G_2 cos(k\omega_0 t) + G_3 sin(k\omega_0 t)), \quad (15)$$

where k is the harmonic index, $G_1 = \{\sin(k\pi\eta + \frac{k\pi}{})\}/k$, $G_2 = \{\sin(\pi\eta k) + \sin(\pi\eta k)\}/k$, and $G_3 = \{\cos(\pi\eta k) - \cos(\pi\eta k)\}/k$. Equations (12) and (13) are, respectively, the positive and negative signal for in-phase LO. Equations (14) and (15) are the positive and negative signal for the quadrature LO, respectively. η is the change in the waveform's duty cycle. Figure 2 shows the $IP2$ metric of the model as function of load and duty cycle imbalance. The outputs of the in-phase mixer and quadrature are, respectively,

$$V_{outI}(t) \quad R_P\,(i_{RF}(t)LO_{PI}(t)) - R_N\,(i_{RF}(t)LO_{NI}(t)), \quad (16)$$

$$V_{outQ}(t) \quad R_P\left(i_{RF}(t)LO_{PQ}(t)\right) - R_N\left(i_{RF}(t)LO_{NQ}(t)\right), (17)$$

where $R_P = R + R\Delta R'$, $R_N = R - R\Delta R'$. R is the expected resis-tance load, and $\Delta R'$ is the percentage of load imbalance. The output of the mixers is $y(t) = V_{outI}(t) + jV_{outQ}(t)$ and can be rewritten as

$$\begin{aligned} y(t) &= \left(\frac{\alpha_2(2\eta + \Delta R')}{2}\right)(|z'_{l,0}(t)|^2 + |z'_{l,1}(t)|^2)(1+j) \\ &+ \left(z'_{l,0}(t) + z'_{l,1}(t)e^{j(\omega_1-\omega_0)t}\right)\left(\frac{G'}{\pi}\right) + \omega(t), \end{aligned} \quad (18)$$

where $G' = \cos(\pi\eta)$. Then, the analog-to-digital converter (ADC)-sampled version of (18) is

$$\begin{aligned} y[n] &= \left(\frac{\alpha_2(2\eta + \Delta R')}{2}\right)(|z'_{l,0}[n]|^2 + |z'_{l,1}[n]|^2)(1+j) \\ &+ \left(z'_{l,0}[n] + z'_{l,1}[n]e^{j(\omega_1-\omega_0)[n]}\right)\left(\frac{G'}{\pi}\right) + \omega[n], \end{aligned} \quad (19)$$

where

$$\begin{aligned} z'_{l,0}[n] &= \alpha_1 z_{l,0}[n] + \frac{3}{4}\alpha_3 z_{l,0}[n]|z_{l,0}[n]|^2 \\ &+ \frac{3}{2}\alpha_3 z_{l,0}[n]|z_{l,1}[n]|^2 \end{aligned} \quad (20)$$

$$\begin{aligned} z'_{l,1}[n] &= \alpha'_1 z_{l,1}[n] + \frac{3}{2}\alpha'_3|z_{l,0}[n]|^2 z_{l,1}[n] \\ &+ \frac{3}{4}\alpha'_3|z_{l,1}[n]|^2 z_{l,1}[n]. \end{aligned} \quad (21)$$

2.3. Signal-to-noise ratio analysis Now, the effects of the distortion components related to the final baseband signal-to-noise ratio (SNR) will be analyzed. The SNR is defined as

$$SNR = \frac{\sigma_s^2}{\sigma_w^2}, \quad (22)$$

where $\sigma_s^2 = E[|s(t)|^2]$ is the signal variance and $\sigma_w^2 = E[|\omega(t)|^2]$

is the noise signal variance. For analysis purposes, $z_{l,0}(t)$ and $z_{l,1}(t)$ are assumed as to be Gaussian circular processes (OFDM modulated signal). SNR_0 can be defined as the SNR in the absence of distortion components, and this is expressed as

$$SNR_0 = \frac{(G'\alpha_1/\pi)^2\sigma_{zl,0}^2}{\sigma_w^2} \quad (23)$$

Finally, the effective SNR $SNR_{effective}$ in presence of interference is expressed by

$$SNR_{effective} = \frac{(G'\alpha_1/\pi)^2\sigma_{zl,0}^2}{\sigma_{wi}^2}, \quad (24)$$

where σ_{wi}^2 is defined as the noise plus interference signal variance.

The procedure to calculate σ_{wi}^2 is explained in Appendix B.

Figures 3 and 4 plot SNR_0 versus $SNR_{effective}$ and show the impact of the blocker signal under some scenarios. The model parameters were set to get results similar to those reported in Ref. [1]. The parameter settings are $\alpha_1 = 56.23$, $\alpha_{2mix} = 0.1$, $\alpha_3 = -7497.33$ and $\sigma^2_{zl,0} = 5 \times 10^{-10}$. It simulates a receiver with

35 dB of linear gain, -20 dB of $IP3$ on the LNA, and 25 dB of $IP2$ on the trasconductance amplifier before the differential mixer. In these figures, $SNR_{effective}$ is plotted as a function of SNR_0 for different values of mixer imbalance (modifying the effective $IP2$). Since a higher LNA gain increases the mixer's $IP2$ requirement [16], the behavior of the model is consistent with the expected behavior on a physical implementation. Figure 3 shows the $SNR_{effective}$ for a blocker signal with $\sigma^2_{z,1} = 0.5 \times 10^{-4}$ (50 dB

greater than the signal of interest) and Fig. 4 shows the $SNR_{effective}$ for a blocker signal with $\sigma^2_{z,1} = 5 \times 10^{-4}$ (60 dB greater than the signal of interest). As can be seen, SNR_0 has high degradation in the presence of the stronger block signal. Also, the influence of third-order nonlinear products on the LNA can be seen, which is evident when the mixer's $IP2$ is infinite (case reported in Ref. [1]). From these figures, it can be deduced that the implementation of a canceler for third- and second-order nonlinear products is required to improve the direct conversion performance in presence of a strong blocker signal.

3. Algorithm to Recover the Blocker Signal Information

This section discusses the available methods to recover the blocker signal information in order to construct the distortion canceler. In the next subsections, two methods are discussed: the first adds an RF circuitry, and the second takes advantage of the band-pass sampling theory properties.

3.1. Compensation by adding an RF circuitry The compensation scheme proposed in Ref. [1] needs a downconverter for every interference signal, as indicated in Fig. 5. The problem is that a downconversion stage is needed for every blocker signal. Another solution was presented in Ref. [11], where the authors proposed the generation of the reference signal in order to train the digital filter with an external analog circuitry and sample reference signal with an additional ADC. Unfortunately, as mentioned previously, any extra RF circuitry is undesirable, especially in deeply integrated systems.

3.2. Compensation by an algorithm based on band-pass sampling Another way to recover the blocker signal information was presented in Ref. [10]. This does not need an additional circuitry but the blocker position is limited to the ADC bandwidth defined by the operating Nyquist frequency.

In order to include within the ADC operating bandwidth a given high-frequency blocker signal, $z_{l,1}(t)e^{j(\omega_1-\omega_0)t}$, and knowing that any solution with a very high sample rate could be impractical, recovering the blocker signal information by means the properties of the band-pass sampling is proposed. First, a sampled version of $z_{l,0}(t)$ and $z_{l,1}(t)$ is needed. From (18), it is noted that $z_{l,0}(t)$ and $z_{l,1}(t)$ are low pass and band pass, respectively. Band-pass sampling theory allows us to subsample a band-pass signal and to exploit the signal information available in their alias spectra. The idea is to get the signal of interest $z_{l,0}(t)$ and one alias of the blocker $z_{l,1}(t)$ within the same ADC bandwidth, both occupying adjacent subbands but without any overlap. A simplified scheme of the proposed algorithm is presented in Fig. 6.

The valid uniform sample rate f_s to undersample a band-pass signal is

$$\frac{2f_u}{n} \leq f_s \leq \frac{f_l}{n-1}, \quad (25)$$

where n must be

$$1 \leq n \leq floor\left(\frac{f_u}{B}\right), \quad (26)$$

f_u is the higher frequency component, f_l is the lower frequency component, and B is the signal bandwidth. Fulfilling (25) ensures reconstructing the band-pass signal without aliasing [22].

In our case, two signals are sampled, and it is necessary to add a guard band to avoid any overlapping between the signal of interest and the blocker alias. In Fig. 7(a), the spectrum centered at $f_{c,zl,1}$ represents the alias with the blocker information $z_{l,1}(t)$. Note that it is a band-pass signal and has a bandwidth of $B_{zl,1}$. The highest frequency component of $z_{l,1}(t)$ is defined by $f_{u,zl,2}$. Fortunately, in a real implementation it is necessary to add a guard band to avoid engineering imperfections. The signal of interest $z_{l,0}(t)$, as baseband, is centered at $f_{c,zl,0} = 0$ and has a bandwidth $B_{zl,0}$. After the sampling process, the signal spectrum can be represented as in Fig. 7(b). The subbands m_i have a bandwidth equal to the selected sample rate f_s, and m_0 is the subband corresponding to the ADC operation spectrum. The dotted spectrum profiles represent the alias spectra due to the sampling process. $m_{zl,1}$ is the subband where the blocker signal $z_{l,1}(t)$ is located. After a convenient band-pass sampling, $f'_{c,zl,1}$ appears in the subband m_0 as the central frequency for the first alias of $z_{l,1}(t)$. $f_{c,zl,1}$ and $B_{zl,1}$ may be obtained by one of the well-known spectrum sensing algorithms, such as the ones proposed in Refs. [23,24]. Hence it is assumed that these parameters are known.

Since the bandwidth of each subband m_i of the spectrum is equal to the sample rate frequency f_s . if f_s is tuned, it is possible to move the first blocker alias to any subband m_i and, as expected, any alias blocker $m_{zl,1}$ is also moved. Our algorithm is focused on finding the lower sample rate to get both the spectra of the signal of interest $z_{l,0}(t)$ and the alias of one blocker signal $z_{l,1}(t)$ adjacent within the ADC operating bandwidth m_0.

As is explained in Ref. [25], band-pass sampling is focused on the baseband of the signal of interest. In our case, to avoid any spectral overlap of $z_{l,0}(t)$ and $z_{l,1}(t)$, it is assumed that the lowest bound of $f'_{c,zl,1}$ is defined by

$$f'_{c,zl,1} \geq \frac{B_{zl,0} + B_{zl,1}}{2}, \quad (27)$$

and the lower bound of f_s is

$$f_s/2 \geq \frac{B_{zl,0}}{2} + B_{zl,1}. \quad (28)$$

One way to avoid the overlapping of $z_{l,1}(t)$ with the signal of interest is defining the upper bound as a function of the negative frequency range of $z_{l,1}(t)$, mirrored from the positive one. Both spectral regions, positive and negative, are defined by

$$|f'_{c,zl,1}| \geq \frac{B_{zl,0} + B_{zl,1}}{2}: \quad (29)$$

where f_s has to be found such that

$$f'_{c,zl,1} + m_{zl,1} f_s = f_{c,zl,1}, \quad (30)$$

departing from (28), if $f_s = f'_{c,zl,1}$, the upper bound of $f'_{c,zl,1}$, is defined by

$$|f'_{c,zl,1}| \leq \frac{f_s - B_{zl,1}}{2}. \quad (31)$$

Thus, considering both lower and upper bounds, expressed by (29) and (31), we obtain

$$\frac{f_s - B_{zl,1}}{2} \geq |f'_{c,zl,1}| \geq \frac{B_{zl,0} + B_{zl,1}}{2}. \quad (32)$$

Substituting (30) in (32) results in

$$\frac{f_s - B_{zl,1}}{2} \geq |f_{c,zl,1} - m_{zl,1} f_s|, \geq \frac{B_{zl,0} + B_{zl,1}}{2}, \quad (33)$$

which reduces the search space for the minimum sample rate, with only two unknown parameters: $m_{zl,1}$ and f_s. The solution for $f'_{c,zl,1}$ is calculated by

$$f'_{c,zl,1} = f_{c,zl,1} - m_{zl,1} f_s. \quad (34)$$

Highly configurable ADC and low-pass filter are required, which have been proposed in the literature [26]. It is necessary, for a convenient attenuation of the blocker signal, to avoid ADC saturation while keeping it detectable with a high SNR. Also, in our proposal it is desirable that both the digital filter coefficients and decimation rates can be programmed in runtime. The conditions to sample more than one blocker signal could be obtained by following the methodology proposed in Ref. [27].

Figure 8 shows the simulated spectrum of the signal before using band-pass sampling and Fig. 9 depicts the simulated spectra after band-pass sampling. It can be noted that ADC conversion recovers the blocker signal information without overlapping the signal of interest. The simulation parameters are $B_{zl,0} = 20 \times 10^6$Hz, $B_{zl,1} = 20 \times 10^6$Hz, $f_{c,zl,1} = 280 \times 10^6$Hz, and $SNR = 40$dB, which after band-pass sampling results in $f_s = 60 \times 10^6$, $m_{zl,1} = 5$, and $f'_{c,zl,1} = -20 \times 10^6$Hz.

Algorithm implementation assumes that a spectrum sensing algorithm has been run before the start of packet on reception. Then the ADC sample rate can be adjusted before the pilot sequence arrives and the interference canceler can be trained during the pilot sequence. The case when the blocker appears in the middle of the packet reception is not covered by this algorithm because it needs to know some sequences to be trained.

4. Proposed Compensation Scheme

Departing from (19), (20) and (21), it is possible to recover the signal of interest and the interference information after AD conversion. The signal of interest can be extracted by using a low-pass filter and interference information by using a high-pass filter and a digital downconverter. The sampled representation of the signal of interest after low-pass filtering can be expressed by

$$\begin{aligned} y_0[n] &= \left(\frac{\alpha_2(2\eta + \Delta R')}{2}\right)(|z_{l,0}[n]|^2 + |z_{l,1}[n]|^2)(1+j) \\ &+ \frac{G'}{\pi}\alpha_1 z_{l,0}[n] + \frac{3G'}{4\pi}\alpha_3 z_{l,0}[n]|z_{l,0}[n]|^2 \\ &+ \frac{3G'}{2\pi}\alpha_3 z_{l,0}[n]|z_{l,1}[n]|^2 + \omega[n]. \end{aligned} \quad (35)$$

The interference signal, as a result of the baseband inter-modulation products generated by a given blocker signal, can be expressed by

$$\begin{aligned} y_1[n] &= \frac{G'}{\pi}\alpha'_1\{z_{l,1}[n]\} + \frac{3H_1}{2\pi}\alpha'_3\{|z_{l,0}[n]|^2 z_{l,1}[n]\} \\ &+ \frac{3G'}{4\pi}\alpha'_3\{|z_{l,1}[n]|^2 z_{l,1}[n]\}. \end{aligned} \quad (36)$$

Considering that $\sigma^2_{zl,0} \ll \sigma^2_{zl,1}$, the interference signal that depends of $z_{l,0}(t)$ can be neglected. Now, the signal of interest can be rewritten as

$$\begin{aligned} y_0[n] &\approx \left(\frac{\alpha_2(2\eta + \Delta R')}{2}\right)(|z_{l,1}[n]|^2)(1+j) \\ &+ \frac{G'}{\pi}\alpha_1 z_{l,0}[n] + \frac{3G'}{2\pi}\alpha_3 z_{l,0}[n]|z_{l,1}[n]|^2 \\ &+ \omega[n]. \end{aligned} \quad (37)$$

Then, the sampled representation of the interference signal is

$$y_1[n] \approx \frac{G'}{\pi}\alpha'_1\{z_{l,1}[n]\} + \omega[n]. \quad (38)$$

Equation (38) cannot be right if self-distortion acquires signif-icance; the effect of self-distortion on blocker signal will be analyzed in the next section. Based on (37), the problem can be expressed as the minimization of the difference between the received data and the transmitted data. The preamble symbol, used in most digital wireless communications, may be used to do this. Thus, the optimization problem is formulated as

$$\min_{\hat{\alpha}_1,\hat{\alpha}_2,\hat{\alpha}_3,\hat{\mathbf{h}}} \sum_{n=0}^{N-1} |y_0[n] \ -\hat{\alpha}_1 z_{l,0}[n] - \hat{\alpha}_2 |z_{l,1}[n]|^2 \\ - \hat{\alpha}_3 z_{l,0}[n]|z_{l,1}[n]|^2|^2, \qquad (39)$$

where $\hat{\alpha}_1$, $\hat{\alpha}_2$, and $\hat{\alpha}_3$ are real constants and their optimal values are given by

$$\hat{\alpha}_1 = \frac{G'}{\pi}\alpha_1, \qquad (40)$$

$$\hat{\alpha}_2 = \frac{\alpha_2(2\eta + \Delta R')}{2} \qquad (41)$$

$$\hat{\alpha}_3 = \frac{3G'}{2\pi}\alpha_3. \qquad (42)$$

Recalling (5) and assuming $y_1[n] = \hat{\alpha_1'} z_{l,1}$, where $\hat{\alpha_1'} = (G'/\pi)\alpha_1'$ is the optimal value, (39) can be rewritten as

$$\min_{\alpha_2'',\alpha_3'',\hat{\mathbf{h}}'} \sum_{n=0}^{N-1} |y_0[n] - \alpha_2''|y_1[n]|^2 \\ -(1+\alpha_3''|y_1[n]|^2)x_{l,0}[n] \star h'[l]|^2, \qquad (43)$$

where $h'[l] = \hat{\alpha_1} h[l]$, and $l = \{0, 1, ..., L-1\}$; L is the filter length and the scaling parameters are defined by

$$\alpha_2'' = \frac{\hat{\alpha_2}}{\hat{\alpha_1'}^2}, \qquad (44)$$

and

$$\alpha_3'' = \frac{\hat{\alpha_3}}{\hat{\alpha_1}\hat{\alpha_1'}^2}. \qquad (45)$$

Finally, There is an optimum filter $h'[n]$ associated with every $\hat{\alpha_2''}$ and $\hat{\alpha_3}$ $''$ combination. The problem is nonlinear andnonconvex. The associated optimal of $h'[n]$ for every fixed pair (α_2'', α_3'') can be calculated by solving

$$\min_{\hat{\mathbf{h}}'} \sum_{n=0}^{N-1} |y_0[n] - \alpha_2''|y_1[n]|^2 \\ -(1+\alpha_3''|y_1[n]|^2)x_{l,0}[n] \star h'[l]|^2, \qquad (46)$$

which can be formulated as a linear least-squares (LS) problem [28]

$$\min_{\mathbf{h}} \|\mathbf{y} - \mathbf{AXh}\|^2 \qquad (47)$$

where $\mathbf{y}$, $\mathbf{A}$, $\mathbf{X}$, and $\mathbf{h}$ are defined as follows:

$$\mathbf{y} = \begin{pmatrix} y_0[0] - \alpha_2''|y_1[0]|^2 \\ y_0[1] - \alpha_2''|y_{l,0}[1]|^2 \\ \vdots \\ y_0[N-1] - \alpha_2'|y_1[N-1]|^2, \end{pmatrix} \qquad (48)$$

$$\mathbf{A} = \text{diag}\{1+\alpha_3''|y_1[0]|^2, 1+\alpha_3''|y_1[1]|^2, \cdots 1+\alpha_3''|y_1[N-1]|^2\}. \tag{49}$$

$$\mathbf{X} = \begin{pmatrix} x_{l,0}[0] & x_{l,0}[N-1] & x_{l,0}[N-L+1] \\ x_{l,0}[1] & x_{l,0}[0] & x_{l,0}[N-L+2] \\ \vdots & & \vdots \\ x_{l,0}[N-2] & x_{l,0}[N-3] & x_{l,0}[N-L-1] \\ x_{l,0}[N-1] & x_{l,0}[N-2] & x_{l,0}[N-L] \end{pmatrix}, \tag{50}$$

$$\mathrm{h} = \begin{pmatrix} h'[0] \\ h'[1] \\ \\ \end{pmatrix} \quad h'[L-1] \tag{51}$$

The well known solution of equation (47) is

$$\hat{\mathbf{h}}_{LS} = (\mathbf{X}^*\mathbf{A}^*\mathbf{A}\mathbf{X})^{-1}\mathbf{X}^*\mathbf{A}^*\mathbf{y} \tag{52}$$

and the residual sum of squares (RSS) is

$$\text{RSS} = \mathbf{y}^*\mathbf{y} - \mathbf{y}^*\mathbf{A}\mathbf{X}(\mathbf{X}^*\mathbf{A}^*\mathbf{A}\mathbf{X})^{-1}\mathbf{X}^*\mathbf{A}^*\mathbf{y} \tag{53}$$

where RSS is a function of α2 and α3 .Thatis

$$\text{RSS} = f_{LS}(\alpha_2'', \alpha_3''). \tag{54}$$

The proposed algorithm to find the optimum values for α_2'' and α_3'' is discussed in the next subsection.

4.1. Heuristic for optimization of α_2'' and α_3''

A differential evolution (DE) algorithm [29,30] was selected as an optimization heuristic to find a good solution in a practical time for α_2'' and α_3''. The DE algorithm is a bio-inspired heuristic developed by evolutionary computing. It has been designed to find good solutions, in polynomial time, for optimization problems. The term *evolutionary* makes reference to the use of operations such as ***mutation*** and *crossover* to make exploration and exploitation in the search space. Because of the low complexity of the DE algorithm, it is very convenient to be executed in a low-performance processor; in our case, it can be executed before the baseband processing. Figure 10 shows the NMSE for α_2'' and α_3'' as a function of SNR_0 achieved by our DE algorithm implementation assuming that the self-interference in the blocker reference signal is negligible.

4.2. Proposed compensation algorithm

Once the optimal values for α_2 and α_3 are found, (37) can be rewritten as

$$\begin{aligned} y_0[n] \quad & (1+j)\alpha_2''|y_1[n]|^2 \\ & + (1+\alpha_3''|y_1[n]|^2)x_{l,0}[n] \star h'[l]) \\ & + \omega[n]. \end{aligned} \tag{55}$$

Then, the proposed canceler for the nonlinear distortion is given by

$$\begin{aligned} z_{l,0}'[n] &= y_0[n] - \alpha_2''(|z_{l,1}[n]|^2)(1+j) \\ & -(1+\alpha_3''|y_1[n]|^2), \end{aligned} \tag{56}$$

where z' $[n]$ is still distorted by the channel response. A channel equalization can be performed, as is used in the frequency domain, by a simple least-squares channel equalizer:

$$\hat{H}_{LS}[i] = \frac{Z_{l,0}'[i]}{X_{l,0}[i]} \tag{57}$$

where i is the subcarrier index, $Z'_{l,0}[i]$ and $X_{l,0}[i]$ are the Fourier transform OFDM symbols of $z'_{l,0}[n]$ and $x_{l,0}[n]$, respectively. Finally, the canceler of the channel response is

$$\hat{X}_{l,0}[i] = \frac{Z'_{l,0}[i]}{\hat{H}_{LS}[i]}. \quad (58)$$

where $h_{l,0}(t)$ is a low-pass equivalent channel response. In this paper, the transmitted complex baseband envelope $x_{l,0}(t)$ is assumed to be an orthogonal frequency division multiplexing (OFDM) signal. For analysis purposes, it is also assumed that the received signal includes only the desired signal plus distortions generated by one blocker signal. Under this last consideration, the signal at the input of the LNA is defined by

5. Computer Simulation and Results

5.1. Signal and model characteristics

Computer sim-ulations were carried out with an OFDM signal composed of 64 subcarriers, 2 guard carriers as the guard band, and a cyclic prefix with length of 16 samples. The variance of the signal of interest was $\sigma^2_{zl,0} = 5 \times 10^{-10}$ and the blocker signal power was $\sigma^2_{zl,1} = 3.0 \times 10^{-4}$. The OFDM symbols were windowed by a Tukey window with a rising edge length equal to the cyclic prefix.

To compare the proposed algorithm's performance with respect to that of algorithm in Ref. [1], the model parameters were set as follows: the LNA gain = 35 dB; LNA $IP3 = -20$ dB, and the mixer transcunductance amplifier $IP2 = 20$ dB; the blocker channel attenuation = 40 dB; the mixer load imbalances $\Delta R' = 0.022$; and $\eta = 0.010$. The channel is modeled as a finite impulse response impulse filter with a length of 4 taps. The taps are independently Rayleigh-distributed with the total power normalized to 1.

5.2. Performance results

Figure 11 shows the plot of bit error rate versus SNR_0 in five different scenarios. The first scenario plot (dots) corresponds to the channel equalizer performance without any nonlinear distortion. It defines the lowest bound for both the reference and proposed algorithms. The second plot (squares) corresponds to the performance of the proposed algorithm. Note that the performance of latter is close to the ideal bit error rate and also to the reference algorithm [1], either in the presence of second- and third-order nonlinear distortion (diamond marks) or in the presence of only third-order nonlinear distortion (asterisks). It is clear that the reference algorithm introduced in Ref. [1] has problems in compensating second-order nonlinear distortion (i.e., it performs well only when third-order nonlinear distortion is present). In contrast, the proposed algorithm shows practically the same performance as the reference algorithm (with only a third-order nonlinear distortion) even in presence of both nonlinear distortions. It can be appreciated that the proposed algorithm has an advantage while canceling two harmful components of nonlinear distortion.

5.3. Self-interference on blocker signal effects

Figure 12 shows the plots of the performance of the algorithm dealing with blocker signals with different powers. Since a strong signal can fall in the strong nonlinear region of the LNA, the effects over the digital canceler are evident in Fig. 12. Five cases were simulated, corresponding to blocker signal variances of $\sigma^2_{z,1} = 6 \times 10^{-4}$, $\sigma^2_{z,1} = 5 \times 10^{-4}$, $\sigma^2_{z,1} = 3 \times 10^{-4}$. $\sigma^2_{z,1} = 1 \times 10^{-4}$, and $\sigma^2_{z,1} = 0.5 \times 10^{-4}$, while the variance of the signal of interest was $\sigma^2_{z,0} = 5 \times 10{-}10$, and same model parameters as in Fig. 11. From Fig. 12, the effect of self-interference on the blocker signal is evident; if the blocker signal falls in strong nonlinear region, the performance of the algorithm is lower, which can affect all state-of-the-art algorithms.

6. Conclusions

An algorithm for the cancellation of nonlinear distortions gen-erated by an out-of-band blocker signal was presented. This algorithm is suitable within the context of direct-conversion software-defined radio, where the analog front end may suffer from the lack of a SAW filter. The improvement on detection per-formance after canceling simultaneously second- and third-order nonlinear distortions was shown and also why it is important to build a canceler for both interferences. Our proposal recovers the blocker signal information by means of subsampling and pure dig-ital signal processing, avoiding additional RF circuitry to solve the problem related to getting the out-of-band blocker information. The proposed technique could be used in radios that are frequently exposed to the presence of blocker signals. The performance of the proposed algorithm was compared with a well-known state-of-the-art algorithm, achieving good performance, even canceling both second- and third-order nonlinear distortions.

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Appendix A

Recalling (4), and using (1), we have

$$\begin{aligned} y_p(t) &= \alpha_1 \mathrm{Re}\left\{z_0(t)e^{j\omega_0 t} + z_1(t)e^{j\omega_1 t}\right\} \\ &+ \alpha_2 \mathrm{Re}\left\{z_0(t)e^{j\omega_0 t} + z_1(t)e^{j\omega_1 t}\right\}^2 \\ &+ \alpha_3 \mathrm{Re}\left\{z_0(t)e^{j\omega_0 t} + z_1(t)e^{j\omega_1 t}\right\}^3. \end{aligned} \quad \text{(A1)}$$

Expanding (A1)

$$\begin{aligned} y_p(t) &= \frac{\alpha_2}{2}|z_0(t)|^2 + \frac{\alpha_2}{2}|z_1(t)|^2 \\ &+ \mathrm{Re}\left\{\left(\alpha_1 z_0(t) + \frac{3}{4}\alpha_3 z_0(t)|z_0(t)|^2 + \frac{3}{2}\alpha_3 z_0(t)|z_1(t)|^2\right)e^{j\omega_0 t}\right\} \\ &+ \mathrm{Re}\left\{\left(\alpha_1 z_1(t) + \frac{3}{4}\alpha_3 z_1(t)|z_1(t)|^2 + \frac{3}{2}\alpha_3 z_1(t)|z_0(t)|^2\right)e^{j\omega_1 t}\right\} \\ &+ \mathrm{Re}\left\{\frac{\alpha_2}{2}[z_0(t)]^2 e^{j2\omega_0 t}\right\} \\ &+ \mathrm{Re}\left\{\frac{\alpha_2}{2}[z_1(t)]^2 e^{j2\omega_1 t}\right\} \\ &\quad \left\{\frac{\alpha_3}{4}[z_0(t)]^3 e^{j3\omega_0 t}\right\} \\ &+ \mathrm{Re}\left\{\frac{\alpha_3}{4}[z_1(t)]^3 e^{j3\omega_1 t}\right\} \\ &+ \mathrm{Re}\left\{\alpha_2 z_0(t) z_1(t) e^{j(\omega_0+\omega_1)t}\right\} \\ &+ \mathrm{Re}\left\{\alpha_2 z_0(t) z_1(t) e^{j(\omega_0-\omega_1)t}\right\} \\ &+ \mathrm{Re}\left\{\frac{3\alpha_3}{4} z_0(t)[z_1(t)]^2 e^{j(\omega_0+2\omega_1)t}\right\} \\ &+ \mathrm{Re}\left\{\frac{3\alpha_3}{4} z_0(t)[z_1^*(t)]^2 e^{j(\omega_0-2\omega_1)t}\right\} \\ &+ \mathrm{Re}\left\{\frac{3\alpha_3}{4} [z_0(t)]^2 z_1(t) e^{j(2\omega_0+\omega_1)t}\right\} \\ &+ \mathrm{Re}\left\{\frac{3\alpha_3}{4} [z_0^*(t)]^2 z_1(t) e^{j(2\omega_0-\omega_1)t}\right\}. \end{aligned} \quad \text{(A2)}$$

If a band-pass filter is used in the output of the LNA, then the output can be rewritten as

$$\begin{aligned} y_p(t) = \mathrm{Re}&\left\{\left(\alpha_1 z_0(t) + \frac{3}{4}\alpha_3 z_0(t)|z_0(t)|^2 + \frac{3}{2}\alpha_3 z_0(t)|z_1(t)|^2\right)e^{j\omega_0 t}\right\} \\ + \mathrm{Re}&\left\{\left(\alpha_1 z_1(t) + \frac{3}{4}\alpha_3 z_1(t)|z_1(t)|^2 + \frac{3}{2}\alpha_3 z_1(t)|z_0(t)|^2\right)e^{j\omega_1 t}\right\}. \end{aligned} \quad \text{(A3)}$$

Finally, we get (A3) which is equivalent to (7) without the additive Gaussian noise.

Appendix B

The power of the interference signal is calculated by

$$\begin{aligned} \sigma_{wi}^2 = E[|&\left(\frac{\alpha_2(2\eta + \Delta R')}{2}\right)\left(|z'_{i,0}[n]|^2 + |z'_{i,1}[n]|^2\right)(1+j) \\ &+ \alpha_1 z_1(t) + \frac{3}{4}\alpha_3 z_1(t)|z_1(t)|^2 + \frac{3}{2}\alpha_3 z_1(t)|z_0(t)|^2 + \omega_p|^2]. \end{aligned} \quad \text{(A4)}$$

Expanding (A4) and using some properties of the expected value, we get

$$\begin{aligned}\sigma_{wi}^2 &= 2\left(\frac{\alpha_2(2\eta+\Delta R')}{2}\right)^2 E\left[\left|\left(|z'_{l,0}[n]|^2+|z'_{l,1}[n]|^2\right)\right|^2\right]\\ &+\frac{9}{16}\alpha_3^2 E\left[\left|z_{l,0}\right|^6\right]\\ &+\frac{9}{4}\alpha_3^2 E\left[\left|z_{l,0}\right|^2\right]E\left[\left|z_{l,1}\right|^4\right]\\ &+\frac{18}{8}\alpha_3^2 E\left[\left|z_{l,0}\right|^4\right]E\left[\left|z_{l,1}\right|^2\right]\\ &+E\left[\left|\omega_p\right|^2\right].\end{aligned} \tag{A5}$$

And expanding A5, we get

$$\begin{aligned}\sigma_{wi}^2 &= 2\left(\frac{\alpha_2(2\eta+\Delta R')}{2}\right)^2 \Big(15|\alpha_1|^3|\alpha_3|E\left[\left|z_{l,0}\right|^4\right]E\left[\left|z_{l,1}\right|^2\right]\\ &+(225/8)|\alpha_1|^2|\alpha_3|^2E\left[\left|z_{l,0}\right|^6\right]E\left[\left|z_{l,1}\right|^2\right]\\ &+(117/2)|\alpha_1|^2|\alpha_3|^2E\left[\left|z_{l,0}\right|^4\right]E\left[\left|z_{l,1}\right|^4\right]\\ &+15|\alpha_1|^3|\alpha_3|E\left[\left|z_{l,0}\right|^2\right]E\left[\left|z_{l,1}\right|^4\right]\\ &+(225/8)|\alpha_1|^2|\alpha_3|^2E\left[\left|z_{l,0}\right|^2\right]E\left[\left|z_{l,1}\right|^6\right]\\ &+(81/4)|\alpha_1||\alpha_3|^3E\left[\left|z_{l,0}\right|^8\right]E\left[\left|z_{l,1}\right|^2\right]\\ &+(1107/16)|\alpha_1||\alpha_3|^3E\left[\left|z_{l,0}\right|^6\right]E\left[\left|z_{l,1}\right|^4\right]\\ &+(1107/16)|\alpha_1||\alpha_3|^3E\left[\left|z_{l,0}\right|^4\right]E\left[\left|z_{l,1}\right|^6\right]\\ &+(81/4)|\alpha_1||\alpha_3|^3E\left[\left|z_{l,0}\right|^2\right]E\left[\left|z_{l,1}\right|^8\right]\\ &+|\alpha_1|^4E\left[\left|z_{l,1}\right|^4\right]\\ &+(81/256)|\alpha_3|^4E\left[\left|z_{l,0}\right|^{12}\right]\\ &+(81/256)|\alpha_3|^4E\left[\left|z_{l,1}\right|^{12}\right]\\ &+|\alpha_1|^4E\left[\left|z_{l,0}\right|^4\right]\\ &+3|\alpha_1|^3|\alpha_3|E\left[\left|z_{l,0}\right|^6\right]\\ &+2|\alpha_1|^4E\left[\left|z_{l,0}\right|^2\right]E\left[\left|z_{l,1}\right|^2\right]\\ &+(405/16)|\alpha_3|^4E\left[\left|z_{l,0}\right|^8\right]E\left[\left|z_{l,1}\right|^4\right]\\ &+(81/16)|\alpha_3|^4E\left[\left|z_{l,0}\right|^{10}\right]E\left[\left|z_{l,1}\right|^2\right]\\ &+(5265/128)|\alpha_3|^4E\left[\left|z_{l,0}\right|^6\right]E\left[\left|z_{l,1}\right|^6\right]\\ &+(27/8)|\alpha_1|^2|\alpha_3|^2E\left[\left|z_{l,0}\right|^8\right]\\ &+3|\alpha_1|^3|\alpha_3|E\left[\left|z_{l,1}\right|^6\right]\\ &+(27/8)|\alpha_1|^2|\alpha_3|^2E\left[\left|z_{l,1}\right|^8\right]\\ &+(27/16)|\alpha_1||\alpha_3|^3E\left[\left|z_{l,1}\right|^{10}\right]\\ &+(405/16)|\alpha_3|^4E\left[\left|z_{l,0}\right|^4\right]E\left[\left|z_{l,1}\right|^8\right]\\ &+(81/16)|\alpha_3|^4E\left[\left|z_{l,0}\right|^2\right]E\left[\left|z_{l,1}\right|^{10}\right]\\ &+(27/16)|\alpha_1||\alpha_3|^3E\left[\left|z_{l,0}\right|^{10}\right]\Big)\end{aligned}$$

where $h_{l,0}(t)$ is a low-pass equivalent channel response. In this paper, the transmitted complex baseband envelope $x_{l,0}(t)$ is assumed to be an orthogonal frequency division multiplexing (OFDM) signal. For analysis purposes, it is also assumed that the received signal includes only the desired signal plus distortions generated by one blocker signal. Under this last consideration, the signal at the input of the LNA is defined by

$$
\begin{aligned}
&+ \quad {}_3^2E \\
&+ \frac{9}{4}\alpha_3^2 E\left[\left|z_{l,0}\right|^2\right] E\left[\left|z_{l,1}\right|^4\right] \\
&+ \frac{18}{8}\alpha_3^2 E\left[\left|z_{l,0}\right|^4\right] E\left[\left|z_{l,1}\right|^2\right] \\
&+ E\left[\left|\omega_p\right|^2\right], \qquad \text{(A6)}
\end{aligned}
$$

where $E\left[|z_{l,x}|^p\right]$, with $x \in \{0, 1\}$, is evaluated by

$$
E\left[|z_{l,x}|^p\right] = \begin{cases} 0 & \text{if } p = 0 \\ \sigma^p (p-1)!!, & \end{cases} \qquad \text{(A7)}
$$

where (.)!! calculates the double factorial.

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